Slope of line PO is G_0

$G_2 \le G_0 \le G_1$ e.g. $G_1 = 0.04, G_2 = -0.03$

Note on the figure: G_1 is positive, G_2 is negative

$$\begin{aligned} H_1 &= x_1 G_1 - x_1 G_0 \\ H_2 &= x_2 G_0 - x_2 G_2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_1 &= \tfrac{H_1}{G_1 - G_0} \\ x_2 &= \tfrac{H_2}{G_0 - G_2} \end{aligned}$$

$x_1 + x_2 + \frac{L}{2} = S$ See previous proof: $X = \frac{L}{2}$

S concept here is different from a horizontal curve

$$\tfrac{L}{2} = S - \left(\tfrac{H_1}{G_1 - G_0} + \tfrac{H_2}{G_0 - G_2}\right) \quad G_0 \in [G_2, G_1] \qquad (1)$$

Let: $x = G_0$

$$f(x) = \tfrac{H_1}{G_1 - x} + \tfrac{H_2}{x - G_2}$$

$$f(x) = H_1 (G_1 - x)^{-1} + H_2 (x - G_2)^{-1} \qquad (2)$$

$$f'(x) = H_1 (G_1 - x)^{-2} - H_2 (x - G_2)^{-2} = 0$$

$$\frac{\sqrt{H_1}}{G_1 - x} = \frac{\sqrt{H_2}}{x - G_2} \quad \text{Note } \frac{\sqrt{H_1}}{G_1 - x} \ne -\frac{\sqrt{H_2}}{x - G_2}$$

$$x\sqrt{H_1} - G_2\sqrt{H_1} = G_1\sqrt{H_2} - x\sqrt{H_2}$$

$$x = \frac{G_1\sqrt{H_2} + G_2\sqrt{H_1}}{\sqrt{H_1} + \sqrt{H_2}}$$

$$f(x) = \frac{H_1(\sqrt{H_1} + \sqrt{H_2})}{G_1\sqrt{H_1} - G_2\sqrt{H_1}} + \frac{H_2(\sqrt{H_1} + \sqrt{H_2})}{G_1\sqrt{H_2} - G_2\sqrt{H_2}}$$

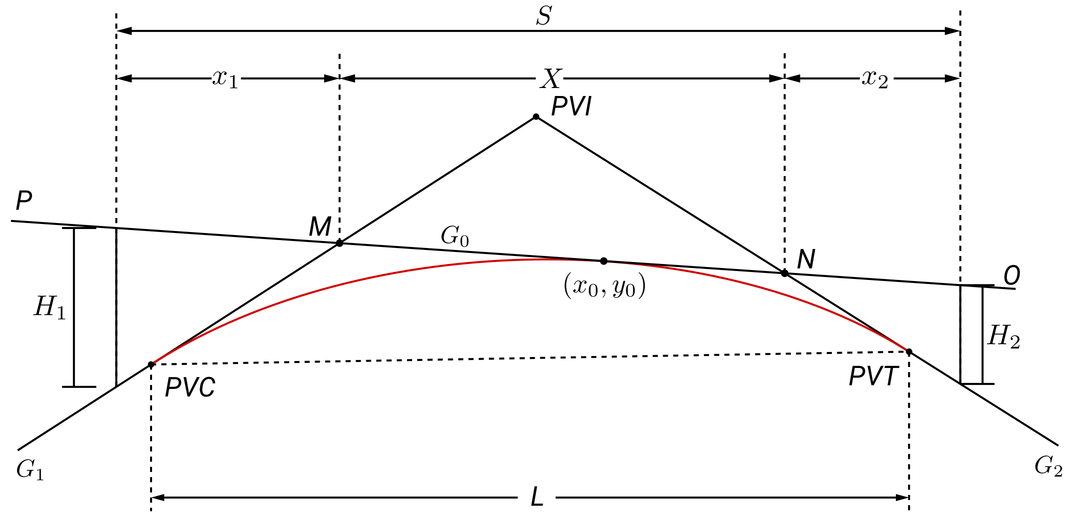
$$f(x) = \frac{\sqrt{H_1}(\sqrt{H_1} + \sqrt{H_2})}{G_1 - G_2} + \frac{\sqrt{H_2}(\sqrt{H_1} + \sqrt{H_2})}{G_1 - G_2}$$

$$f(x) = \frac{(\sqrt{H_1} + \sqrt{H_2})^2}{G_1 - G_2} \quad (*)$$

(1) becomes:

$$L = 2S - \frac{2(\sqrt{H_1} + \sqrt{H_2})^2}{G_1 - G_2} \quad (3)$$

Note on Green Book, Grades are percentages.



PVC – Point of Vertical Curve
PVI – Point of Vertical Intersection
PVT – Point of Vertical Tangent

$$L = 2S - \frac{2(\sqrt{H_1} + \sqrt{H_2})^2}{G_1 - G_2}$$

7th Edition Green Book, Page 3-166

Note from (2)

$$f''(x) = 2H_1(G_1 - x)^{-3} + 2H_2(x - G_2)^{-3}$$

$$x \in [G_2, G_1] \;\Rightarrow\; f''(x) > 0$$

$(*)$ is the minimum value.

L value in equation (3) is the maximum required to provide adequate sight distance.
When an engineer chooses a value for L, the value in equation (3) is the minimum to pick.

Highway Geometric Design - Min Crest Vertical Curve Length