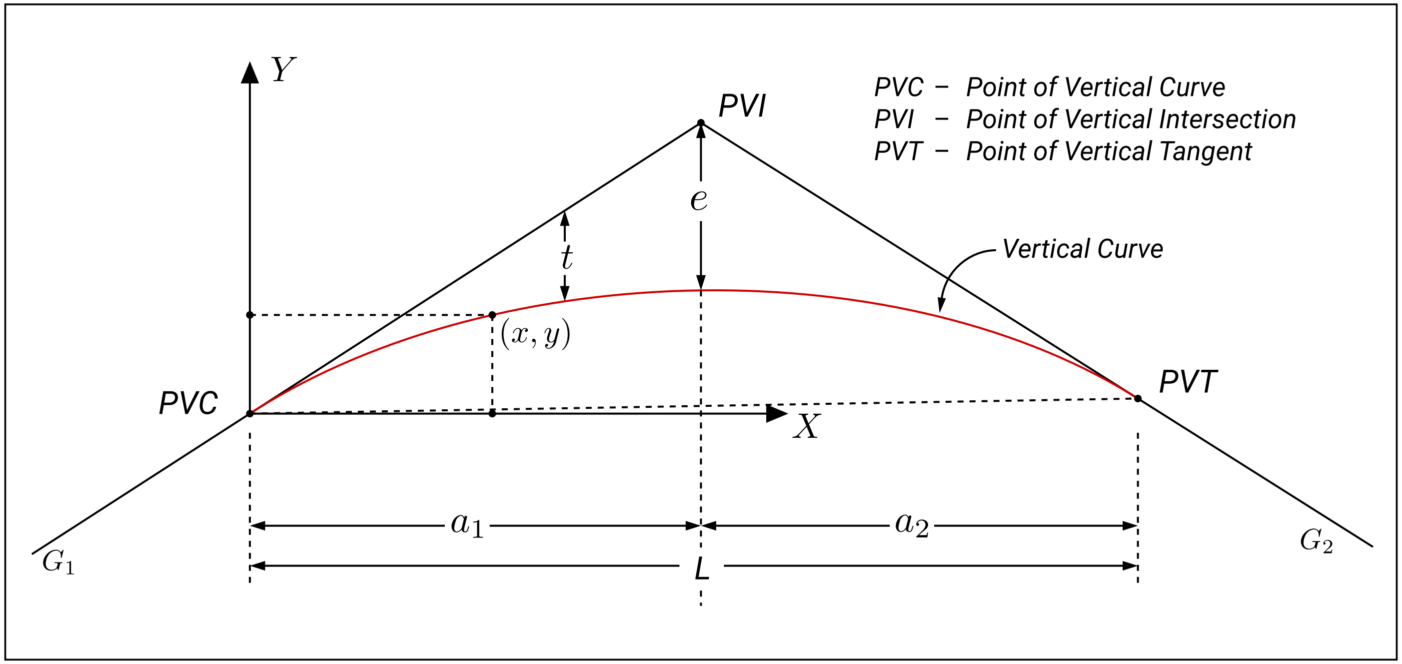

$$y = \frac{-G_1 + G_2}{2L}x^2 + G_1 x \quad (1)$$

$$y' = \frac{-G_1 + G_2}{L}x + G_1 \quad (2)$$

Prove that $a_1 = a_2 = \frac{L}{2}$

$$x = L \quad y = a_1 G_1 + a_2 G_2$$

$$a_1 G_1 + a_2 G_2 = \frac{L}{2}(G_1 + G_2)$$

$$a_1 + a_2 = L$$

$$a_1 G_1 + (L - a_1) G_2 = \frac{L}{2}(G_1 + G_2)$$

$$a_1 (G_1 - G_2) = \frac{L}{2}(G_1 - G_2)$$

$$a_1 = \frac{L}{2}$$

So, $a_1 = a_2 = \frac{L}{2}$

Vertical curve is a parabola:

$$y = ax^2 + bx + c$$

$$y' = 2ax + b$$

$$x = 0, y = 0 \quad \Rightarrow \quad c = 0$$

$$x = 0, y' = G_1 \quad \Rightarrow \quad b = G_1$$

$$y = ax^2 + G_1 x$$

$$y' = 2ax + G_1$$

$$x = L, y' = G_2 \quad \Rightarrow \quad 2aL + G_1 = G_2$$

$$\Rightarrow \quad a = \frac{-G_1 + G_2}{2L}$$

(G1 is positive, G2 is negative,
e.g. G1 = 0.04, G2 = -0.03)

$$x = L, y = -\frac{L}{2}G_1 + \frac{L}{2}G_2 + G_1 L = \frac{L}{2}G_1 + \frac{L}{2}G_2 \qquad \text{PVT}: (L, \frac{L}{2}G_1 + \frac{L}{2}G_2)$$

$$x = L/2 \quad \Rightarrow \quad y = \frac{-G_1 + G_2}{2L} \cdot \frac{L^2}{4} + G_1 \frac{L}{2}$$

$$e = \frac{L}{2}G_1 - \left(\frac{-G_1 + G_2}{2L} \cdot \frac{L^2}{4} + G_1 \frac{L}{2}\right) = \frac{(G_1 - G_2)L}{8} \quad (3)$$

$$A = G_1 - G_2 \quad (4)$$

$$t(x) = xG_1 - y(x) = xG_1 - \left(\frac{-G_1 + G_2}{2L}x^2 + xG_1\right) = \frac{G_1 - G_2}{2L}x^2$$

The highest point on the curve is (x_0, y_0)

$$y' = \frac{-G_1 + G_2}{L}x_0 + G_1 = 0 \quad \Rightarrow \quad x_0 = \frac{LG_1}{G_1 - G_2}$$

$$y_0 = \frac{-G_1 + G_2}{2L}{x_0}^2 + G_1 x_0 = -\frac{L{G_1}^2}{2(G_1 - G_2)} + \frac{L{G_1}^2}{G_1 - G_2} = \frac{L{G_1}^2}{2(G_1 - G_2)}$$

Highway Geometric Design - Vertical Curve Equations