The equation of a vertical curve is

$$y = \frac{-G_1 + G_2}{2L}x^2 + G_1 x$$

$$y' = \frac{-G_1 + G_2}{L}x + G_1 \quad \Rightarrow \quad x = 0, y' = G_1 \quad x = L, y' = G_2$$

The equation of a line is $y = ax + b$

The equation of the line from PVC to PVI is $y = G_1 x$

The equation of the line from PVI to PVT is $y = G_2 x + b_0$

PVI is $(x = \frac{L}{2}, y = G_1 \frac{L}{2})$

PVI is on the line from PVI to PVT, so

$$G_1 \tfrac{L}{2} = G_2 \tfrac{L}{2} + b_0 \quad \Rightarrow b_0 = (G_1 - G_2)\tfrac{L}{2}$$

So the line from PVI to PVT becomes $y = G_2 x + (G_1 - G_2)\frac{L}{2}$

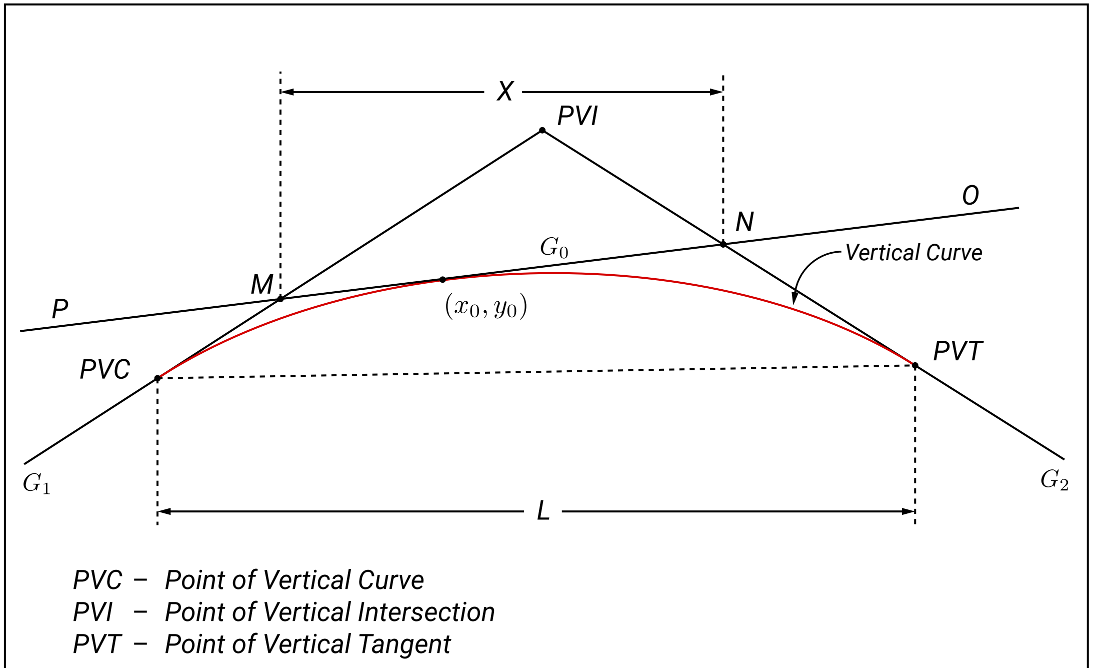
The equation of a line PO is $y = G_0 x + b_1$ assuming the slope is G_0

The tangent point is (x_0, y_0)

$$y' = G_0 = \frac{-G1 + G_2}{L} \cdot x_0 + G_1$$

$$x_0 = -\frac{(G_0 - G_1)L}{G_1 - G_2} \qquad y_0 = \frac{-G_1 + G_2}{2L}x_0^2 + G_1 x_0$$

$$b_1 = y_0 - G_0 x_0 = \frac{-G_1 + G_2}{2L}x_0^2 + (G_1 - G_0)x_0 \quad = -\frac{(G_0 - G_1)^2 L}{2(G_1 - G_2)} + \frac{(G_1 - G_0)^2 L}{G_1 - G2} = \frac{(G_0 - G_1)^2 L}{2(G_1 - G_2)}$$



Point M $\quad y_m = G_0 x_m + b_1 = G_1 x_m$

$$x_m = \frac{b_1}{G_1 - G_0}$$

Point N $\quad y_n = G_0 x_n + b1 = G_2 x_n + \frac{L(G_1 - G_2)}{2}$

$$x_n = -\frac{b_1}{G_0 - G_2} + \frac{L(G_1 - G_2)}{2(G_0 - G_2)} \qquad X = x_n - x_m = \frac{L}{2}\left(\frac{G_1 - G_2}{G_0 - G_2} - \frac{(G_0 - G_1)^2}{(G_0 - G_2)(G_1 - G_2)} - \frac{(G_0 - G_1)^2}{(G_1 - G_0)(G_1 - G_2)}\right) = \frac{L}{2}(*)$$

The denominator in (*) is $(G_0 - G_1)(G_1 - G_2)(G_0 - G_2)$

The numerator in (*) is $(G_0 - G_1)(G_1 - G_2)^2 - (G_0 - G_1)^2(G_0 - G_1) + (G_0 - G_1)^2(G_0 - G_2)$

$$= (G_0 - G_1)(G_1^2 + G_2^2 - 2G_1G_2 - G_0^2 - G_1^2 + 2G_0G_1 + G_0^2 - G_0G_2 - G_0G_1 + G_1G_2)$$

$$= (G_0 - G_1)(G_2^2 - G_1G_2 + G_0G_1 - G_0G_2) \quad = (G_0 - G_1)(G_0 - G_2)(G_1 - G_2)$$

So: $X = \frac{L}{2}$

Highway Geometric Design - Vertical Curve Property